A Mortar Mimetic Finite Difference Method on Nonmatching Grids

Markus Berndt, Konstantin Lipnikov, and Mikhail Shashkov, (T-7); M.F. Wheeler (University of Texas); I. Yotov (University of Pittsburg); ***berndt@lanl.gov***

Mimetic finite difference methods (MFD) are gaining popularity for the discretization of many types of partial differential equations. In practical applications it is often desired to discretize different regions of the physical domain with regular grids and then patch these regions together. The resulting discretization is then referred to as a mortar method.

Our focus in this work is on analyzing MFD approximations to the mixed form of second-order elliptic problems on nonmatching quadrilateral and triangular multiblock grids. To this end we employ mortar finite elements on the nonmatching interfaces to impose weak continuity of the velocity. We show optimal convergence and, for certain cases, superconvergence is established for both the pressure and the velocity (all the details can be found in [1]).

The mortar mixed finite element (mortar MFE) method is well known and has been studied, for example, in [2, 3]. In this method, the domain is divided into nonoverlapping subdomain blocks, and each of these subdomain blocks is discretized on a locally constructed mesh. As a result, the subdomain grids do not match at interblock boundaries. To solve this problem, Lagrange multiplier pressures are introduced at the interblock boundaries.

This Lagrange multiplier space is called the mortar finite element space. It was shown in [3] that the mortar MFE method is optimally convergent, if the boundary space has one order higher approximability than the normal trace of the velocity space.

A connection between the mimetic finite difference (MFD) method and the MFE method with Raviart-Thomas finite elements was established in [4]. This was achieved by showing that the scalar product in the velocity space proposed in [5] for MFD methods can be viewed as a quadrature rule in the context of MFE methods. In [6], superconvergence for the normal velocities in MFD methods on h^2-uniform quadrilateral meshes is established.

The graph in the figure shows convergence rates for the pressure and velocity on a sequence of randomly perturbed multiblock quadrilateral meshes of the type that is

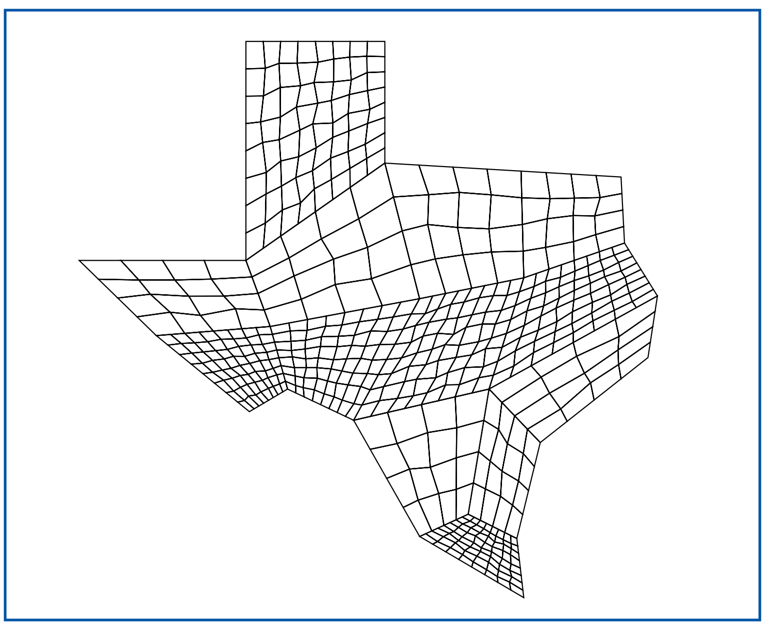

Figure 1*—*
A randomly perturbed multiblock quadrilateral mesh.

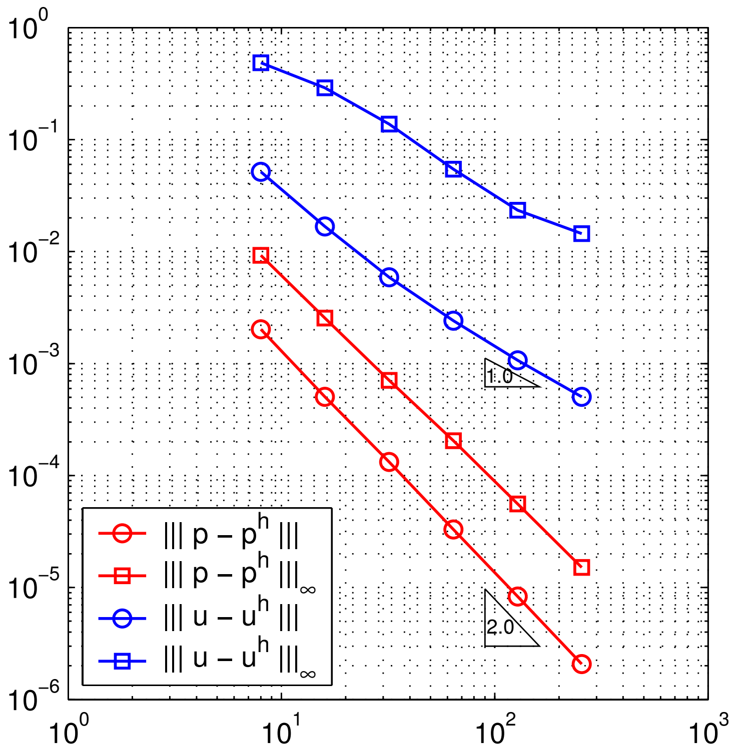

Figure 2—
A convergence study on a sequence of meshes using our mortar finite difference method.

depicted above it. The mortar MFD method with our special scalar product has been used on this random mesh. The asymptotically optimal convergence rate for the velocity is in agreement with our theory. The second-order convergence rate for the pressure variable was also theoretically predicted.

[1] M. Berndt, K. Lipnikov, M. Shashkov, M.F. Wheeler, and I. Yotov, "A Mortar Mimetic Finite Difference Method on Nonmatching Grids", submitted to *Numerische Mathematik,* 2004.
[2] I. Yotov, "Mixed Finite Element Methods for Flow in Porous Media," Ph.D. thesis, Rice University (1996).
[3] T. Arbogast, L.C. Cowsar, M.F. Wheeler, and I. Yotov, "Mixed Finite Element Methods on Nonmatching Multiblock Grids," *SIAM J. Numer. Anal.*, **37** (4):1295–1315, 2000.
[4] M. Berndt, K. Lipnikov, J.D. Moulton, and M. Shashkov, "Convergence of Mimetic Finite Difference Discretizations of the Diffusion Equation," *East-West J. Numer. Math.*, **9**:253–284, 2001.
[5] J.M. Hyman, M. Shashkov, and S. Steinberg, "The Numerical Solution of Diffusion Problems in Strongly Heterogeneous Nonisotropic Materials," *J. Comput. Phys.*, **132**:130–148, 1997.
[6] M. Berndt, K. Lipnikov, M. Shashkov, M.F. Wheeler, and I. Yotov, "Superconvergence of the Velocity in Mimetic Finite Difference Methods on Quadrilaterals," submitted to *SIAM J. Numer. Anal.*, 2004.